Geometrical Model for Massive MIMO Systems

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***Abstract*—Recently, massive multiple-input multiple-output (MIMO) systems have attracted considerable research interest and have been regarded as a candidate technology for the 5th generation (5G) cellular networks. In massive MIMO systems, a base station (BS) is equipped with a large number of antennas which are serving several simultaneous single antenna users. However, it is difficult to place an increasing number of antennas in a limited space, and the antenna space limitation causes a high spatial correlation between the antennas, further resulting in systems performance degradation. In this paper, we establish a 3-D geometrical channel model for massive MIMO systems. We focus on the massive MIMO system antennas average correlation and channel capacity. The far-field and near-field effects are modeled by the plane wave (PW) and spherical wave (SW), respectively. We derive average correlation for a certain antenna to describe the certain one antenna correlation degree in the whole massive MIMO system, and define average correlation for the whole system antennas to describe the whole system antennas correlation degree.**

***Index Terms*—Massive MIMO, 5G, spatial correlation, channel model.**

I. Introduction

In recent years, massive multiple-input multiple-output (MIMO) systems have attracted considerable research interest, which have been regarded as a candidate technology for the 5th generation (5G) cellular networks. Generally speaking, a massive MIMO system can be considered as an enhanced version of conventional MIMO system by utilizing an enormous number of antennas [1]. But unlike conventional MIMO systems with small and compact antenna arrays, massive MIMO systems have a base station (BS) equipped with a large number of antennas (dozens or hundreds of antennas) serving several simultaneous single antenna users.

As the number of antennas increases, Liu et al. [2] pointed out that when the distance between the transmitter and receiver is less than the Rayleigh distance due to the huge size of the massive antenna structure, the plane wave (PW) condition is dissatisfied, hence it is necessary to consider spherical wave (SW). Most researches on the massive MIMO systems are based on the assumption that the channels are independent. In reality, this assumption is very difficult to realize when the number of multiple antennas is very large [3]. An antenna spacing of at least half a wavelength at the customer premise equipment and ten wavelengths at the base station are typically required for achieving significant channel capacity [4]. However, it is difficult to place an increasing number of antennas in a limited space, and if it can be deployed, the high spatial correlation between the antennas will cause system performance degradation [5].

In this paper, we establish a 3-D geometrical channel model for massive MIMO systems, and both the azimuth angles of arrival (AAoAs) and elevation angles of arrival (EAoAs) have been taken into account. We focus on the massive MIMO system antennas average correlation and channel capacity. We use PW to model the far-field signals, and use SW to model the near-field signals. The average correlation for a certain antenna is derived to describe the certain one antenna correlation degree in the whole massive MIMO system, and average correlation for the whole system antennas is defined to describe the whole system antennas correlation degree. The channel capacity has also been derived and analyzed. The results show that near-field effects help decorrelating the antennas, and the average correlation for both a certain antenna and the whole system antennas decrease to a certain extent as the number of antennas increases. The SW effect is weakened as the distance between transmitter and receiver increases, and the SW can be regarded as PW when the transmitter and receiver is far enough apart.

The remainder of this paper is organized as follows. Section II analyzes the uniform linear array (ULA) massive MIMO system antennas average correlation and channel capacity in both far-field using PW and near-field using SW. Section III gives the simulation results and analysis. Finally, we present the concluding remarks in Section IV.

II. Analysis of Massive MIMO System Antennas Average Correlation and Channel Capacity

In this section, we analyze the average correlation for both a certain antenna and the whole uniform linear array (ULA) massive MIMO system antennas. Both the far-field effect using PW and near-field effect using SW have been modeled, and the channel capacity has been derived and analyzed.

A. Massive MIMO System Antennas Average Correlation Using PW

A 3-D ULA massive MIMO system transmission scenario using PW is shown in Fig. 1. The antennas are in the far-field of the signals (more than the Rayleigh distance), therefore, the signals at antennas are parallel, which only have transmission distance differences. The x-S_1-y plane is the horizontal plane. The multipath signals come from arbitrary direction with AAoAs $\alpha_n (n \in \{1, 2...N\})$ and EAoAs $\beta_n (n \in \{1, 2...N\})$

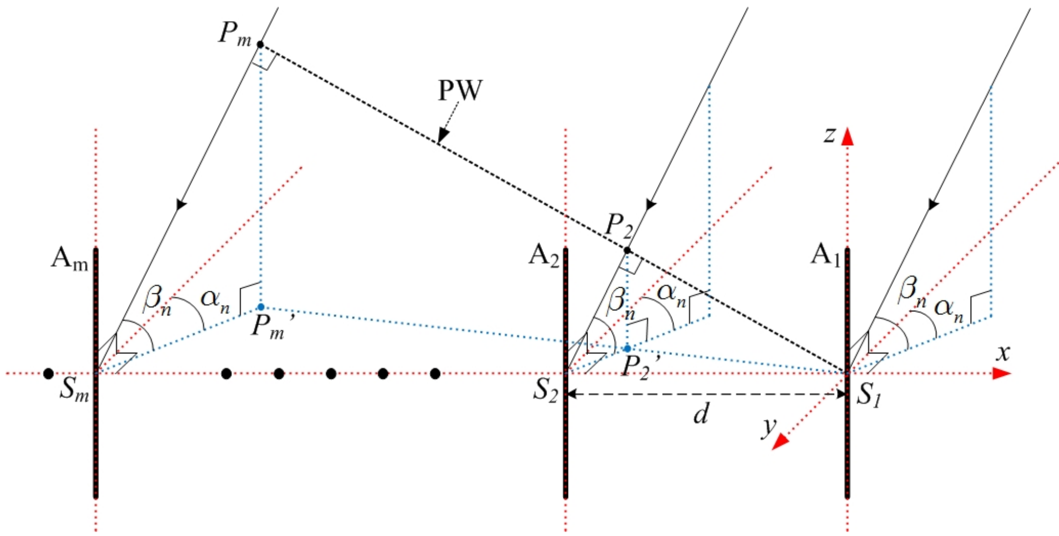


Fig. 1. 3-D ULA massive MIMO system transmission scenario using PW.

for all the antennas, and every antenna has the same AAoAs and EAoAs distributions. The adjacent antenna spacing is d. For antenna A$_2$, $S_2P_2^{'}$ is the projection of S_2P_2 on the x-S_1-y plane. $\angle P_2S_2P_2^{'}$ is β_n, and $\angle P_2^{'}S_2S_1$ is equal to $(\pi/2-\alpha_n)$. Therefore, the angle $\angle P_2S_2S_1$ is given by

$$\begin{aligned} cos\angle P_2S_2S_1 &= cos\angle P_2^{'}S_2S_1 cos\angle P_2S_2P_2^{'} \\ &= cos(\pi/2-\alpha_n)cos(\beta_n) \end{aligned} \quad (1)$$

where (1) is proved in the appendix. The difference of transmission distance when wave front arrives at antenna A$_1$ and antenna A$_2$ respectively is P_2S_2, and the corresponding time delay difference is P_2S_2/c=d$cos(\pi/2-\alpha_n)cos(\beta_n)$/c. Regarding antenna A$_1$ as a reference antenna, the channel impulse responses at antenna A$_1$ and antenna A$_2$ using PW can be expressed as

$$h_1^{PW} = \sum_{n=1}^{N} A_n e^{j\phi_n} \quad (2)$$

$$h_2^{PW} = \sum_{n=1}^{N} A_n e^{j(\phi_n + 2\pi d cos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda)} \quad (3)$$

where A_n is the nth path receiving amplitude and ϕ_n is the nth path receiving phase. λ is the carrier wavelength. In massive MIMO systems, all the antennas are assumed to be uniformly-spaced, and for PW all the signals are parallel, therefore, for antenna A$_m$, the channel impulse response is

$$h_m^{PW} = \sum_{n=1}^{N} A_n e^{j(\phi_n + 2\pi d(m-1)cos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda)} \quad (4)$$

And the spatial correlation function between antenna A$_1$ and antenna A$_2$ using PW is defined as $\rho_{1,2}^{PW}$

$$\begin{aligned} \rho_{1,2}^{PW} &= E\{h_1^{PW*}h_2^{PW}\} \\ &= E\{\sum_{n=1}^{N} A_n^2 e^{-j2\pi d cos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda}\} \end{aligned} \quad (5)$$

As $N \rightarrow \infty$, discrete AAoAs α_n and discrete EAoAs β_n can be replaced by continuous random variables α and β having a joint probability density function (pdf) $p(\alpha,\beta)$ [6]. We assume that AAoAs and EAoAs are independent of each other, then we have $p(\alpha,\beta)=p(\alpha)p(\beta)$. Then $\rho_{1,2}^{PW}$ can be written as

$$\rho_{1,2}^{PW} = A^2 \int\int e^{-j2\pi d cos(\frac{\pi}{2}-\alpha)cos(\beta)/\lambda} p(\alpha)p(\beta)d\beta d\alpha \quad (6)$$

where A is the receiving amplitude at the antennas. The spatial correlation function between any two antennas is

$$\begin{aligned} \rho_{m,k}^{PW} &= E\{h_m^{PW*}h_k^{PW}\} \\ &= A^2 \int\int e^{-j2\pi d(k-m)cos(\frac{\pi}{2}-\alpha)cos(\beta)/\lambda} \\ & p(\alpha)p(\beta)d\beta d\alpha \end{aligned} \quad (7)$$

Since the massive MIMO systems have dozens or hundreds of antennas, we define the average correlation for a certain antenna to describe the certain one antenna correlation degree in the whole massive MIMO system. We assume that the whole ULA massive MIMO system has M antennas, and for antenna A$_1$, the average correlation $\bar{\rho}_1^{PW}$ is defined as

$$\begin{aligned} \bar{\rho}_1^{PW} &= \frac{\rho_{1,1}^{PW}+\rho_{1,2}^{PW}+...+\rho_{1,M}^{PW}}{M} \\ &= \frac{\sum_{i=1}^{M}\rho_{1,i}^{PW}}{M} \end{aligned} \quad (8)$$

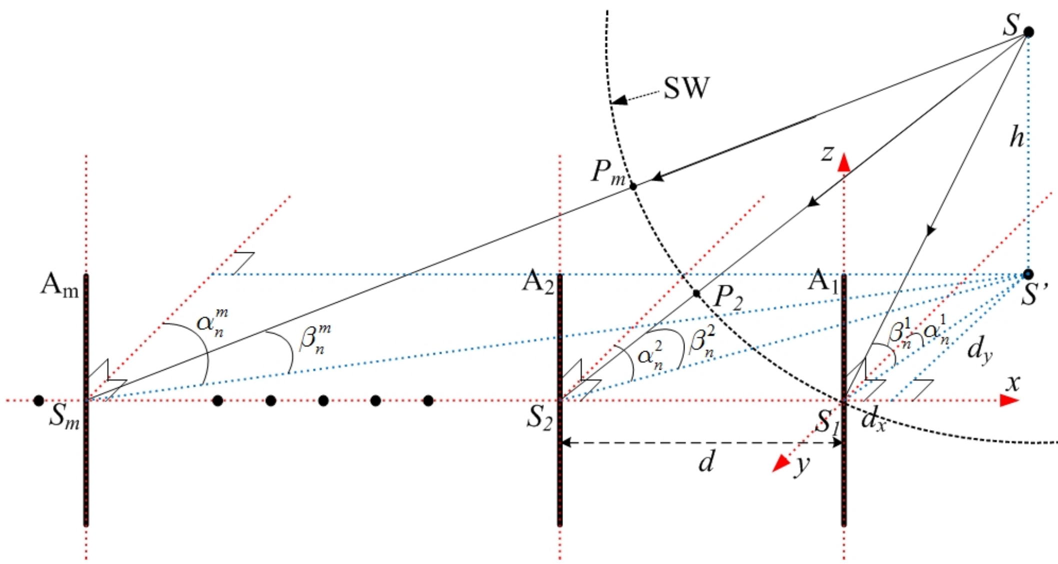


Fig. 2. 3-D ULA massive MIMO system transmission scenario using SW.

And for antenna A_m, the average correlation $\bar{\rho}_m^{PW}$ is

$$\begin{aligned}\bar{\rho}_m^{PW} &= \frac{\rho_{m,1}^{PW} + \rho_{m,2}^{PW} + ... + \rho_{m,M}^{PW}}{M} \\ &= \frac{\sum_{i=1}^{M} \rho_{m,i}^{PW}}{M} \end{aligned} \tag{9}$$

The average correlation of a certain antenna is used to describe the correlation degree of the certain one antenna. However, for the whole massive MIMO system, we define the average correlation for the whole system antennas to describe the whole massive MIMO system antennas correlation degree, which is actually the mean value of all the average correlation of a certain antenna

$$\begin{aligned}\bar{\rho}^{PW} &= [(\rho_{1,1}^{PW} + \rho_{1,2}^{PW} + ... + \rho_{1,M}^{PW}) + \\ &(\rho_{2,1}^{PW} + \rho_{2,2}^{PW} + ... + \rho_{2,M}^{PW}) + \\ &... + (\rho_{M,1}^{PW} + \rho_{M,2}^{PW} + ... + \rho_{M,M}^{PW})]/M^2 \\ &= \frac{\sum_{i=1}^{M} \bar{\rho}_i^{PW}}{M} \end{aligned} \tag{10}$$

The antenna correlation is influenced by the environment scattering (i.e., AAoAs and EAoAs distributions). There are many different distributions to characterize the AAoAs distribution, such as uniform, Gaussian, Laplacian. Usually, we use the uniform distribution with certain angle spread (AS) to characterize the AAoAs distribution, which is defined as

$$p(\alpha) = \frac{1}{2\Delta\alpha}, -\Delta\alpha + \alpha_0 \le \alpha \le \Delta\alpha + \alpha_0 \tag{11}$$

where α_0 is the mean AAoAs, and $\Delta\alpha$ is the AS. For the EAoAs distribution, we use the cosine probability density function [7]

$$p(\beta) = \frac{\pi}{4\beta_{max}} \cos(\frac{\pi}{2}\frac{\beta}{\beta_{max}}), |\beta| \le \beta_{max} \le \frac{\pi}{2} \tag{12}$$

where β_{max} is the maximum EAoA.

B. Massive MIMO System Antennas Average Correlation Using SW

As mentioned above, when the distance between the transmitter and receiver is less than the Rayleigh distance, the PW condition is dissatisfied, and then we have to consider SW. Fig. 2 shows the 3-D ULA massive MIMO system transmission scenario using SW. The only difference between Fig. 2 and Fig. 1 is that the wave front is sphere rather than planar. We assume that the signals come from signal source S, and $S^{'}$ is the projection of S on the horizontal plane x-S_1-y. The distance between the $S^{'}$ and the x axis is d_y, the distance between the $S^{'}$ and the y axis is d_x, and the height of S is h. Then we can obtain the exact distances between the signal source S and every antenna

$$\begin{aligned} d_{SS_1} &= \sqrt{d_x^2 + d_y^2 + h^2} \\ d_{SS_2} &= \sqrt{(d_x + d)^2 + d_y^2 + h^2} \\ &... \\ d_{SS_m} &= \sqrt{(d_x + (m-1)d)^2 + d_y^2 + h^2} \end{aligned} \tag{13}$$

Also we regard antenna A_1 as a reference antenna, and the channel impulse responses of different antennas using SW are

$$\begin{aligned} h_1^{SW} &= \sum_{n=1}^{N} A_n e^{j(\phi_n + 2\pi\sqrt{d_{x,n}^2 + d_{y,n}^2 + h_n^2}/\lambda)} \\ h_2^{SW} &= \sum_{n=1}^{N} A_n e^{j(\phi_n + 2\pi\sqrt{(d+d_{x,n})^2 + d_{y,n}^2 + h_n^2}/\lambda)} \\ &... \\ h_m^{SW} &= \sum_{n=1}^{N} A_n e^{j(\phi_n + 2\pi\sqrt{(d(m-1)+d_{x,n})^2 + d_{y,n}^2 + h_n^2}/\lambda)} \end{aligned} \tag{14}$$

TABLE I

COMPARISON OF MEAN AS UNDER DIFFERENT COMMUNICATION ENVIRONMENTS.

Frequency (MHz)	Urban	Suburban	Countryside	Indoor
1000				$20^\circ \sim 60^\circ$
1800	8°	5°		
1845			$<10^\circ$	
1873	$3^\circ \sim 15^\circ$			
2100	$7^\circ \sim 12^\circ$	$3^\circ \sim 18^\circ$		
2154			10.3°	
2200			3°	
7000				$22^\circ \sim 26^\circ$

According to the geometrical relationship shown in Fig. 2, for an arbitrary antenna we have

$$\begin{aligned} tan(\alpha_n^m) &= \frac{d(m-1)+d_{x,n}}{d_{y,n}} \\ tan(\beta_n^m) &= \frac{h_n}{\sqrt{(d(m-1)+d_{x,n})^2+d_{y,n}^2}} \end{aligned} \tag{15}$$

And then we have

$$\begin{aligned} d_{x,n} &= tan(\alpha_n^m) * dy,n - d(m-1) \\ h_n &= tan(\beta_n^m)\sqrt{tan^2(\alpha_m) * d_{y,n}^2 + d_{y,n}^2} \end{aligned} \tag{16}$$

The spatial correlation between any two different antennas using SW is

$$\begin{aligned} \rho_{m,k}^{SW} &= E\{h_m^{SW*} h_k^{SW}\} \\ &= A^2 \int\int e^{j\frac{2\pi}{\lambda}[\sqrt{(d(m-1)+d_x)^2+d_y^2+h^2}} \\ & e^{-j\frac{2\pi}{\lambda}\sqrt{(d(k-1)+d_x)^2+d_y^2+h^2}]} p(\alpha)p(\beta)d\beta d\alpha \end{aligned} \tag{17}$$

Also we can define the average correlation of a certain antenna as $\bar{\rho}_i^{SW}$ and that of the whole ULA massive MIMO system antennas as $\bar{\rho}^{SW}$ when PW is replaced by SW.

C. Massive MIMO System Channel Capacity

Assuming that a typical massive MIMO system with a BS equipped with M antennas are serving K ($K < M$) simultaneous single antenna users. The K single antenna users are at random positions in the same cell. The channel capacity is highly dependent on the correlation between antennas (both the transmit antennas correlation and receive antennas correlation). The generalized channel capacity can be computed as [8]

$$\mathbf{C} = \log_2[\det(\mathbf{I} + \frac{\rho}{\mathrm{M}}\mathbf{H}\mathbf{H}^{\mathrm{H}})] \tag{18}$$

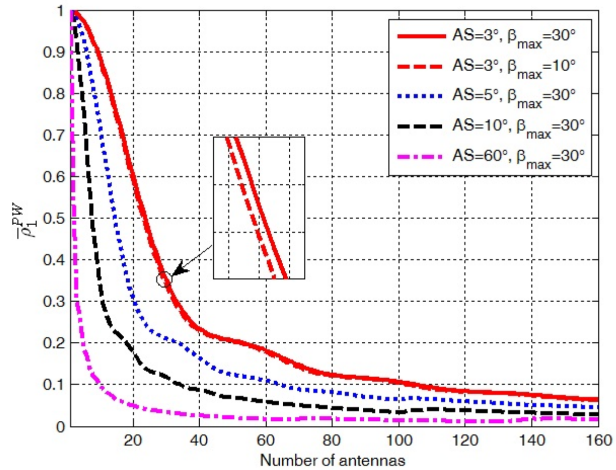
where $\mathbf{I}$ is the $K \times K$ identity matrix, $\mathbf{H}$ denotes the $K \times M$ channel matrix and $\mathbf{H}^H$ is its conjugate transpose. ρ is the average signal-to-noise ratio (SNR) at each receiver branch [9]. The channel matrix $\mathbf{H}$ can be modeled as Kronecker model [10]

$$\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{H}_{\mathbf{i.i.d.}}\mathbf{R}_T^{T/2} \tag{19}$$

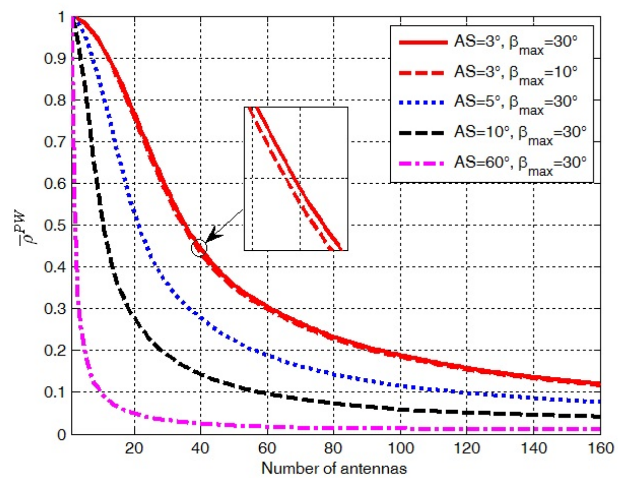
where $\mathbf{H}_{\mathbf{i.i.d.}}$ is a $K \times M$ matrix of independent and identically distributed (i.i.d.) zero mean complex-valued Gaussian random variables. $\mathbf{R}_R$ is $K \times K$ receive correlation matrix and $\mathbf{R}_T$ is $M \times M$ transmit correlation matrix. $(\cdot)^T$ denotes the transpose operation. For massive MIMO system, we only consider the correlation at the BS because the user antennas are uncorrelated since they are practically very distant, hence $\mathbf{R}_R = \mathbf{I}$ [11]. The elements of matrix $\mathbf{R}_T$ are obtained using (7) and (17).

III. SIMULATION RESULTS AND ANALYSIS

Table I [12] summarizes the mean AS under different real communication environments, and we set our simulation parameters according to the real scenarios. From Fig. 3(a) we can see the average correlation of a certain antenna (here we consider the first antenna A_1) using PW varies as the number of antennas increases with different AS. The antenna spacing is equal to $\lambda/2$, $\beta_{max} = 30^\circ$ and the amplitude is normalized. We observe that the average correlation is sensitive to the AS while the EAoAs distribution has a slight effect on the correlation since it is placed in the horizontal plane, and the larger AS results in the lower correlation. When the number of antennas is small, the large AS (i.e., the rich scattering) results in a very low average correlation, which has a great advantage compared with the small AS. As the number of antennas increases, the advantage of the average correlation with a large AS is weakened. For instance, when the number of antennas is 20, the average correlation of the certain one antenna with AS=3° is about 0.6, while the average correlation with AS=60° is lower than 0.1. As the number of antennas increases, the average correlation of the certain one antenna decreases. When the number of antennas is 160, the average correlation when AS=3° is also lower than 0.1, which has little difference with the average correlation value with AS=60°. This is because when the number of antennas is small, all the antennas are very close to each other (may be several wavelengths), so the antennas need rich scattering to reduce the average correlation. But when the number of antennas is large, more antennas are far from the certain one antenna, therefore, the average correlation of a certain antenna becomes low as the number of antennas increases. Fig. 3(b) shows the average correlation of the whole ULA massive MIMO system antennas using PW. Also we can see as the number of antennas increases, the advantage of the average correlation with a large AS is weakened. Therefore, massive MIMO systems do not need rich scattering (large AS) to decrease the correlation compared to the traditional MIMO systems. In addition, the

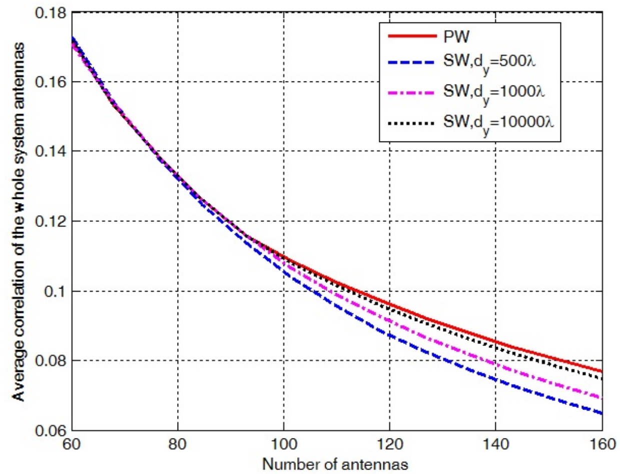


(a)

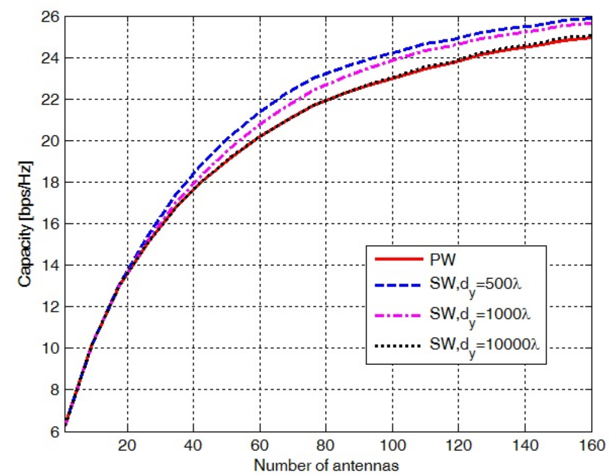


(b)

Fig. 3. (a) $\bar{\rho}_1^{PW}$ versus number of antennas using PW with different AS (b) $\bar{\rho}^{PW}$ versus number of antennas using PW with different AS.



(a)



(b)

Fig. 4. (a) The whole system antennas average correlation comparison of PW and SW versus number of antennas (b) Channel capacity comparison of PW and SW versus number of antennas.

average correlation for both a certain antenna and the whole system antennas decrease to a certain extent as the number of antennas increases, and keeping further increasing the number of antennas makes little improvement.

Fig. 4(a) shows the average correlation of the whole massive MIMO system antennas comparison of using PW and SW with a signal source at different distances. The AS is set to be a typical value 5°, from which we can see that when d_y is small (i.e., the distance between the transmitter and receiver is small), the average correlation using SW is lower than that using PW, which means that the SW effect is obvious and the near-field effects help decorrelating the antennas. While when d_y becomes larger (i.e., the distance between the transmitter and receiver becomes larger), the SW gets more closer to PW, which means the SW effect is weakened when the distance between transmitter and receiver increases. Fig. 4(b) shows the corresponding channel capacity comparison of using PW and SW with a signal source at different distances, where SNR is set to be 10 dB and $K = 8$. The channel capacity increases as the number of antennas increases, and the lower average correlation results in the higher channel capacity. Also when d_y is small the channel capacity using SW is higher than using PW because of the lower average correlation. While when d_y becomes larger, the SW gets closer to PW. In addition, the SW curve overlaps the PW curve when the d_y is large enough, then the SW can be regarded as PW when the transmitter is far enough away from the receiver. For both Fig. 4(a) and Fig. 4(b), when the number of antennas is small, all the curves overlap because of the small antennas size. As the the number of antennas increases, the differences between SW and PW appear due to the massive antenna structure.

IV. Conclusion

In this paper, a 3-D geometrical channel model is established for massive MIMO systems. We consider both far-field effect using PW and near-field effect using PW. We focus on the massive MIMO system antennas average correlation and channel capacity. The average correlation for a certain antenna is derived to describe the certain one antenna correlation degree, and the average correlation for the whole system antennas is defined to describe the whole system antennas correlation degree. The near-field effects help decorrelating the antennas, and the average correlation for both a certain antenna and the whole system antennas decrease to a certain extent as the number of antennas increases. The SW effect is weakened when the distance between the transmitter and the receiver increases, and the SW can be regarded as PW when the transmitter is far enough away from the receiver.

Appendix

Proof of equation (1)

In Fig. 5, l is a straight line in the plane ε. Line segment OA intersects plane ε at point A. B is the projection of O on plane ε. Line segment BC intersects with l at point C at a right angle. We define $\angle OAC = \vartheta, \angle BAC = \vartheta_1, \angle OAB = \vartheta_2$.

$$\because OB \perp \varepsilon$$

$$\therefore OB \perp AC$$

$$\because AC \perp BC$$

$$\therefore AC \perp OBC,\ AC \perp OC$$

And we have:

$$cos\vartheta = \frac{AC}{AO},\ cos\vartheta_1 = \frac{AC}{AB},\ cos\vartheta_2 = \frac{AB}{AO}. \tag{20}$$

$$cos\vartheta_1 cos\vartheta_2 = \frac{AC}{AB} \cdot \frac{AB}{AO} = \frac{AC}{AO} = cos\vartheta \tag{21}$$

$$cos\vartheta = cos\vartheta_1 cos\vartheta_2 \tag{22}$$

Acknowledgement

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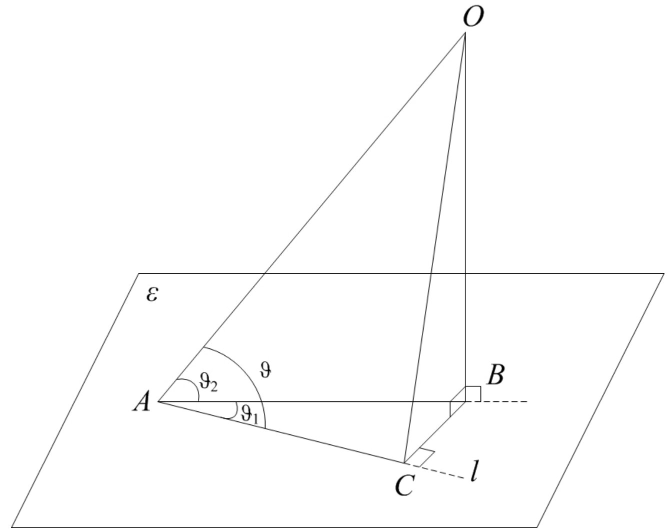

Fig. 5. The diagram of three concurrent angles.

References

[1] Shangbin Wu, Cheng-Xiang Wang, Harald Haas, el-Hadi M. Aggoune, Mohammed M. Alwakeel and Bo Ai, "A Non-Stationary Wideband Channel Model for Massive MIMO Communication Systems," *IEEE Transactions on Wireless Communications*, vol. 14, no. 3, Mar. 2015, pp. 1434-1446.

[2] Liu Liu, David W. Matolak, Cheng Tao, Yongzhi Li, Bo Ai and Houjin Chen, "Channel capacity investigation of a linear massive MIMO system using spherical wave model in LOS scenarios," *Science China Information Sciences*, vol. 59, no. 2, Feb. 2016, pp. 1-15.

[3] Xin Fang, Sheng Fang, Na Ying, Haiyan Cao and Chao Liu, "The performance of massive MIMO systems under correlated channel," *Proc. 2013 19th IEEE International Conference on Networks (ICON)*, Dec. 2013, pp. 1-4.

[4] Claude Oestges, Maxime Guillaud and Merouane Debbah, "Multi-Polarized MIMO Communications Channel Model, Mutual Information and Array Optimization," *Proc. 2007 IEEE Wireless Communications and Networking Conference(WCNC)*, Mar. 2007, pp. 1057-1061.

[5] Jaehyun Park and Bruno Clerckx, "Multi-User Linear Precoding for Multi-Polarized Massive MIMO System Under Imperfect CSIT," *IEEE Transactions on Wireless Communications*, vol. 14, no. 5, May 2015, pp. 2532-2547.

[6] Seok-Chul Kwon and Gordon L. Stüber, "Geometrical Theory of Channel Depolarization," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 8, Oct. 2011, pp. 3542-3556.

[7] J. D. Parsons and A. M. D. Turkmani, "Characterisation of mobile radio signals: Model description," *Proc. IEE I - Communications, Speech and Vision*, vol. 138, no. 6, Dec. 1991, pp. 549-556.

[8] Yejun He, Xudong Cheng and Gordon L. Stüber, "On polarization channel modeling," *IEEE Wireless Communications*, vol. 23, no. 1, Feb. 2016, pp. 80-86.

[9] Foschini G. J. and Gans M. J., "On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas," *Wireless Personal Communications*, vol. 6, no. 3, 1998, pp. 311-335.

[10] Shangbin Wu, Cheng-Xiang Wang, El-Hadi M. Aggoune and Mohammed M. Alwakeel, "A novel Kronecker-based stochastic model for massive MIMO channels," *Proc. 2015 IEEE/CIC International Conference on Communications in China (ICCC)*, April 2016, pp. 1-6.

[11] Xavier Artiga, Bertrand Devillers and Julien Perruisseau-Carrier, "Mutual coupling effects in multi-user massive MIMO base stations," *Proc. 2012 IEEE International Symposium on Antennas and Propagation*, July 2012, pp. 1-2.

[12] Yong Soo Cho, Jaekwon Kim, Won Young Yang and Chung G. Kang, *MIMO-OFDM Wireless Communications with MATLAB,* Wiley-IEEE Press, Aug. 2010.